Arrival Curves for Real-Time Calculus: the Causality Problem and its Solutions

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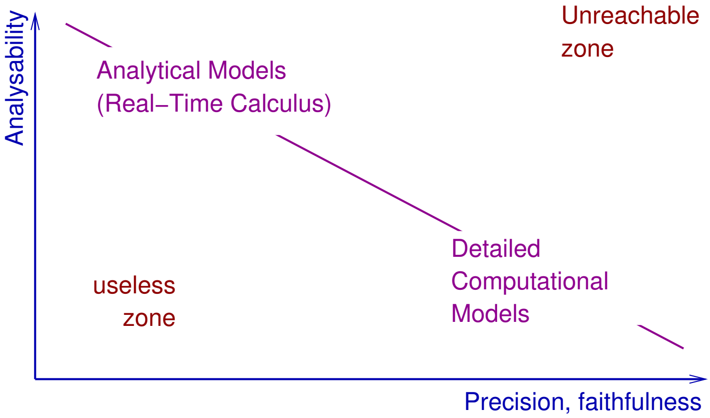
TACAS, 25 March 2010

Models for Performance Analysis



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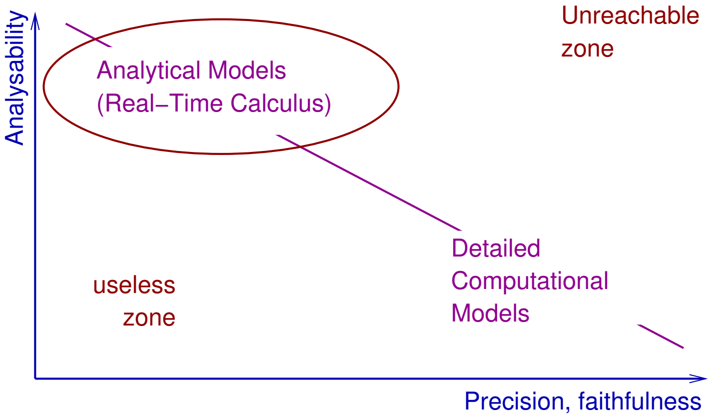

Summary

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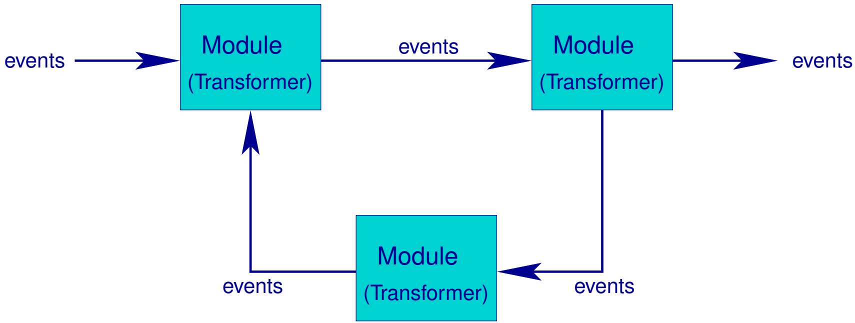
1. Introduction: Modular Performance Analysis
2. The Causality Problem for Arrival Curves
3. The Causality Closure: Solving the Causality Problem
4. Conclusion

Modular Performance Analysis (MPA): The Big Picture



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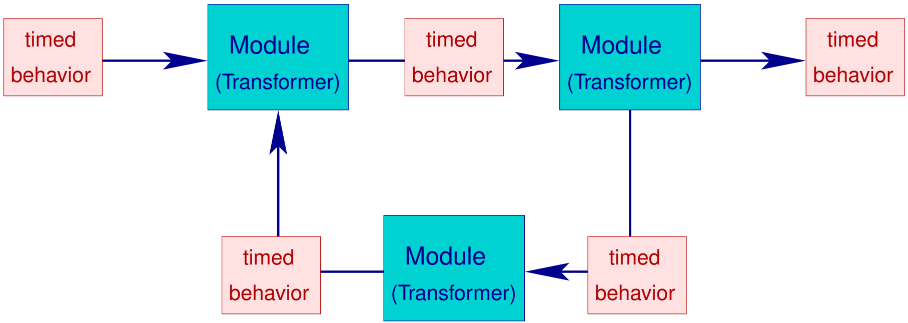

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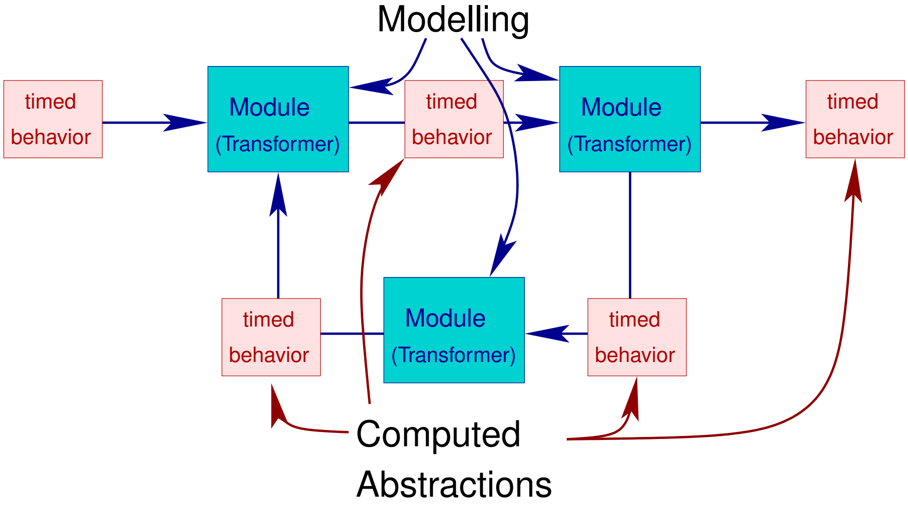

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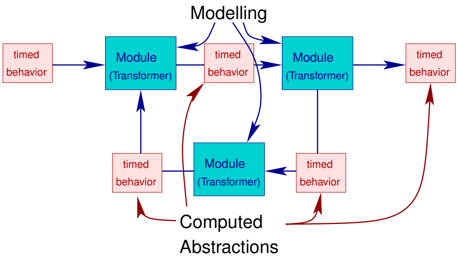

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  - Bounds for number of events?

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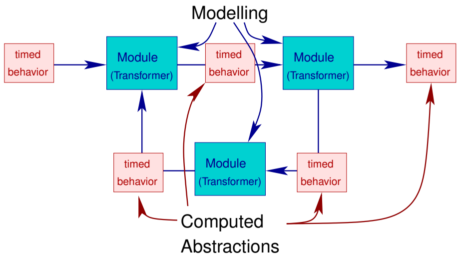
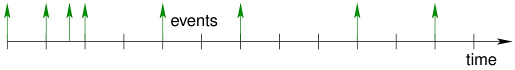


- What can "Timed Behavior" be?
  - Number of events per time unit?
  - Bounds for number of events?
  - MPA uses "Arrival Curves".
- "Modules" = Arrival Curve transformers:
  - FIFO + processing element (defined by "service curves")
  - Can also be a "program"

Arrival Curves in Real-Time Calculus (RTC)



- $\alpha^u(\delta)$: max number of events in any window of size δ.
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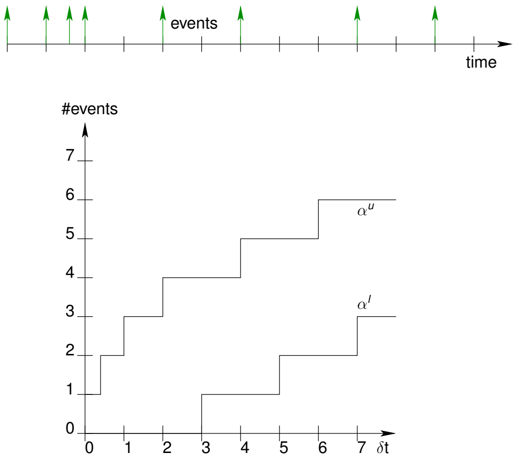

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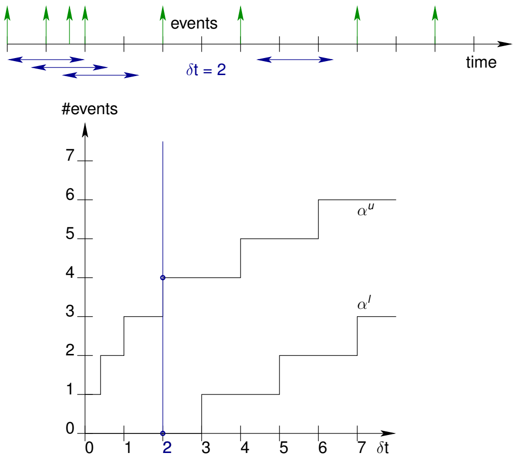

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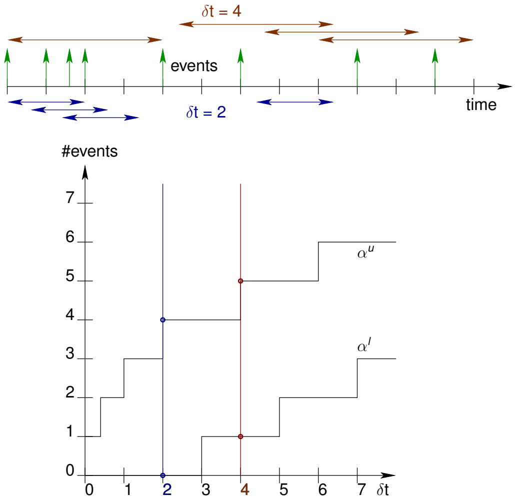

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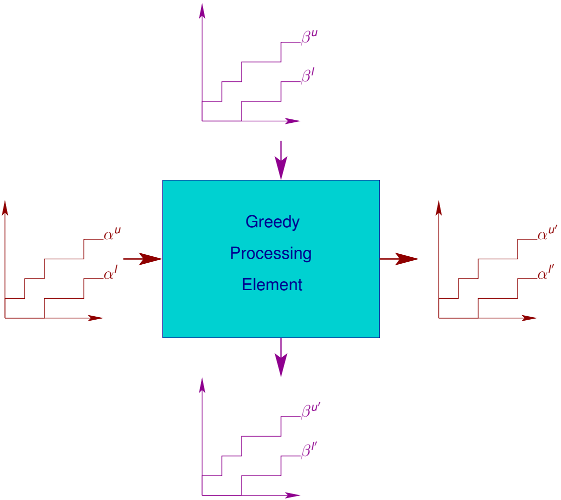

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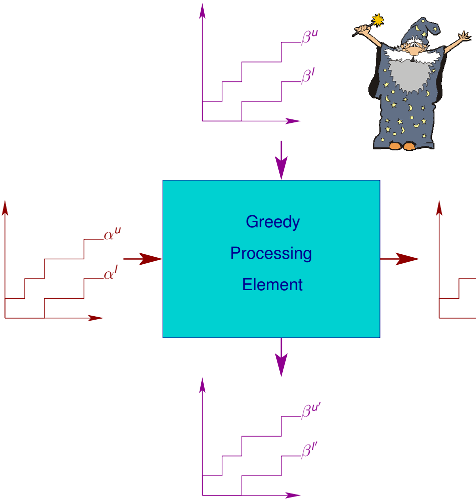

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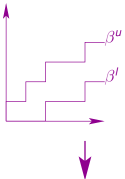
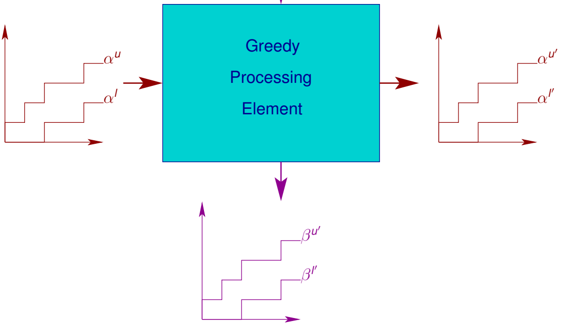


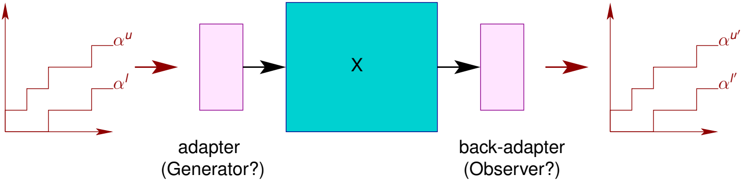


$$\alpha^{u'} = ((\alpha^u \underline{\oplus} \beta^u) \overline{\oplus} \beta^l) \wedge \beta^u$$
$$\alpha^{l'} = ((\alpha^l \overline{\otimes} \beta^u) \underline{\otimes} \beta^l) \wedge \beta^l$$
$$\beta^{u'} = (\beta^u - \alpha^l) \underline{\otimes} 0$$
$$\beta^{l'} = (\beta^l - \alpha^u) \overline{\otimes} 0$$

Real-Time Calculus (RTC): pros and cons

- Nice things with RTC
  - Can model: event streams, simple scheduling policies
  - Scales up nicely
  - Exact hard bounds
- Less nice thing with RTC
  - Limited expressiveness

Allowing more Complex Modules in MPA



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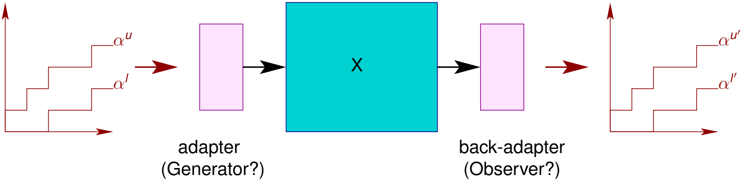
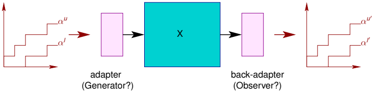


- X = Arbitrary program ⇒ testing (ETHZ)
- X = Timed Automata ⇒ model-checking (Verimag, ETHZ, Uppsala)
- X = Lustre ⇒ Abstract Interpretation, SMT Solving (Verimag)

Summary

A Closer Look at the Generator



- The idea behind generators:

  "at each point in time, compute an interval $[l, u]$ on the number of events that can be emitted".

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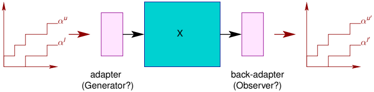

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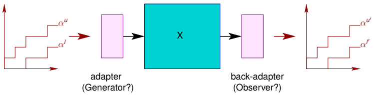

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This talk:

How can we prevent these deadlocks?

Causal and Non-Causal Curves

- A pair of arrival curve (α^u, α^l) is causal iff an event stream conformant with (α^u, α^l) *up to time t* can be extended into a stream conformant with (α^u, α^l) *forever*.

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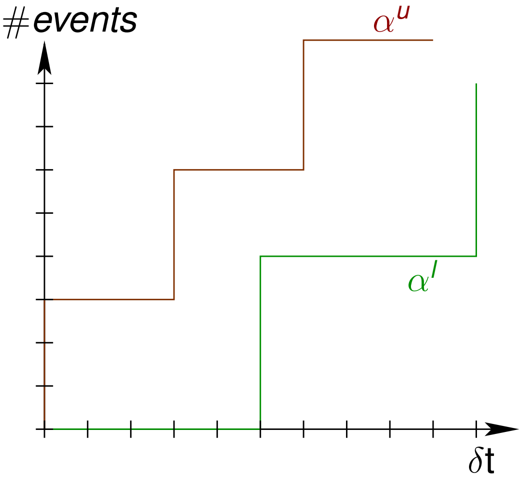
  an event stream conformant with (α^u, α^l) *up to time t* can be extended into a stream conformant with (α^u, α^l) *forever*.
- i.e., (α^u, α^l) is causal iff
  the associated generator has no deadlock.

Causality problem: Forbidden Regions



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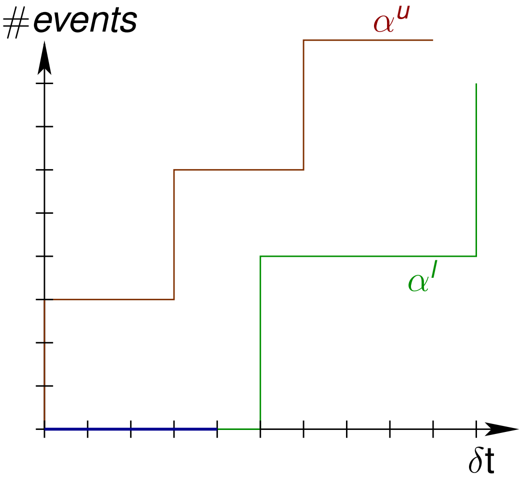

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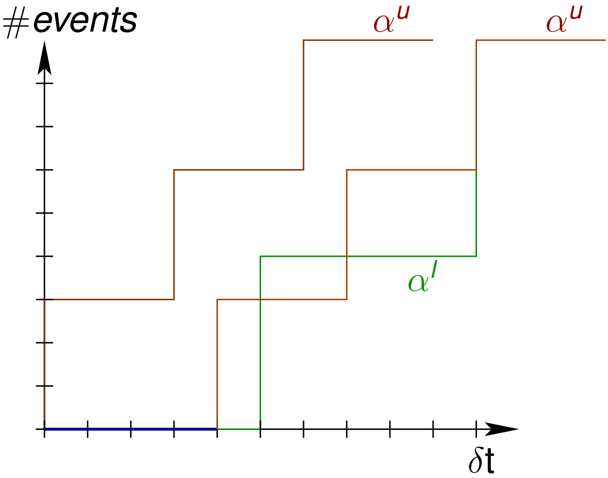

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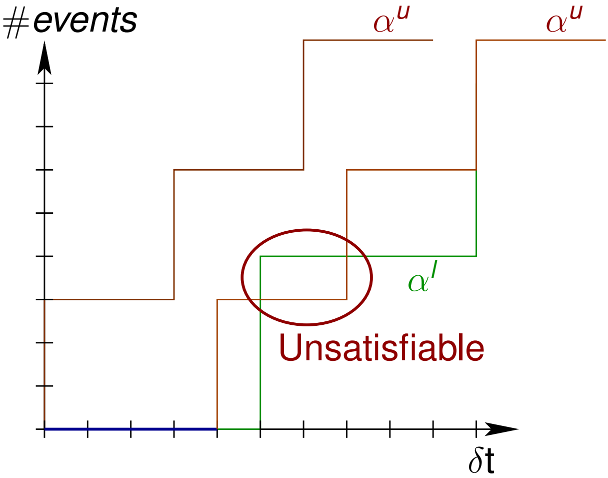

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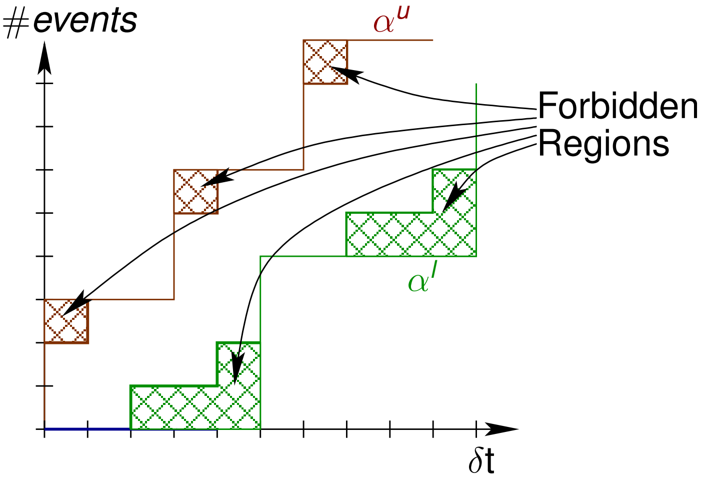

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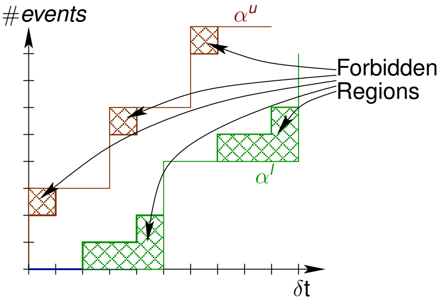

If a stream gets in a forbidden region,
it will eventually reach a dead-end

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(Side question:
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$\rightsquigarrow$ Possible answer at end of talk.

Summary

Causality Closure: Making Curves Causal

- The causality closure of (α^u, α^l) is a pair of curves that is:
  - Equivalent to (α^u, α^l) (i.e. same set of accepted event streams)
  - Causal (i.e. finite accepted event streams can be extended infinitely)
- How to compute it?
- Intuition: Causal curves are curves without forbidden regions (?)

Towards an Algorithm for Causality Closure

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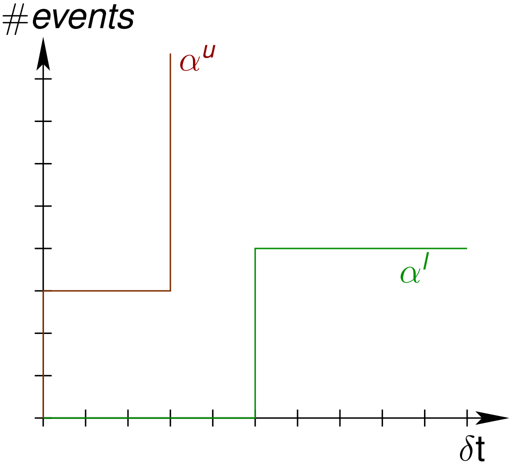

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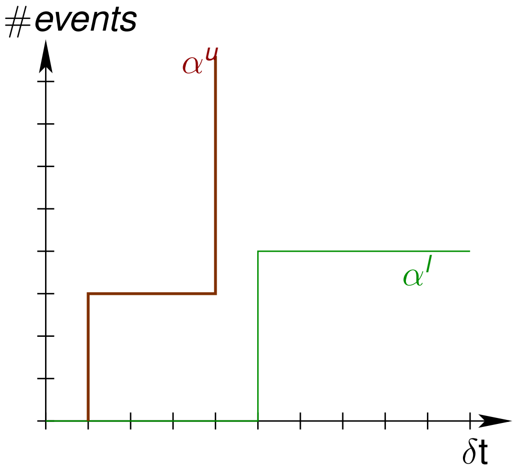

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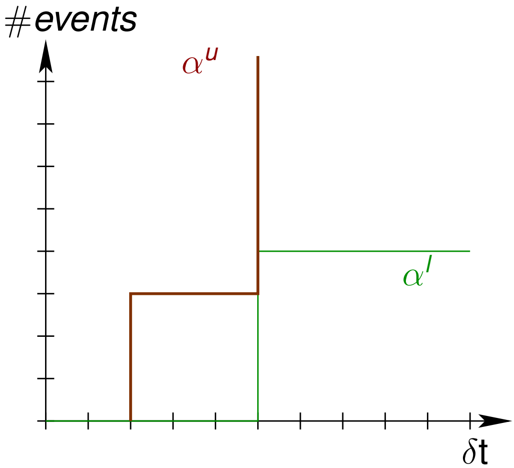

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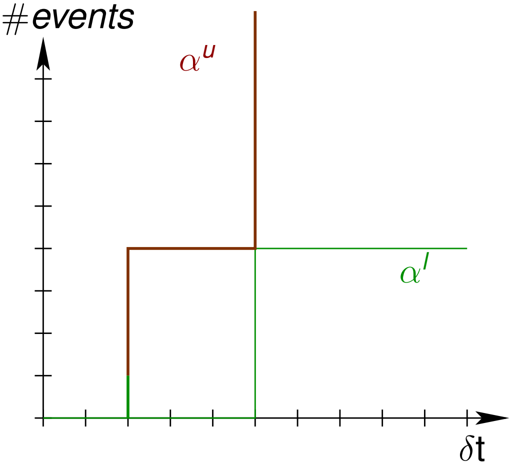

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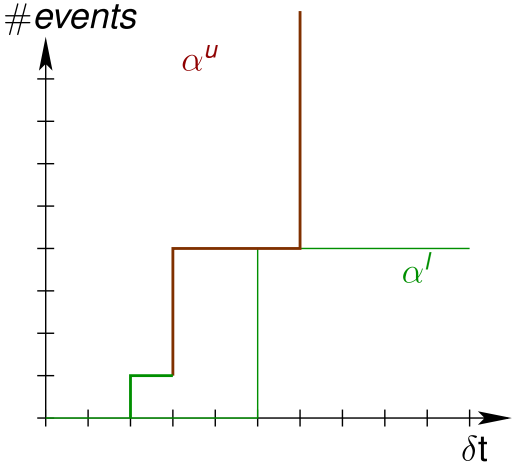

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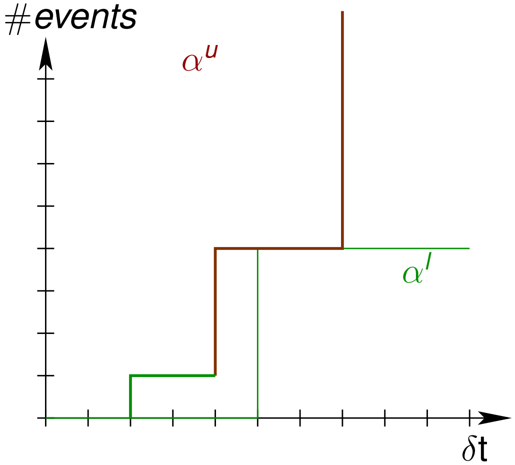

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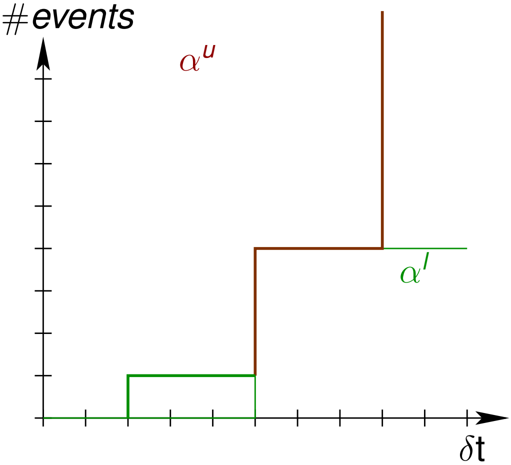

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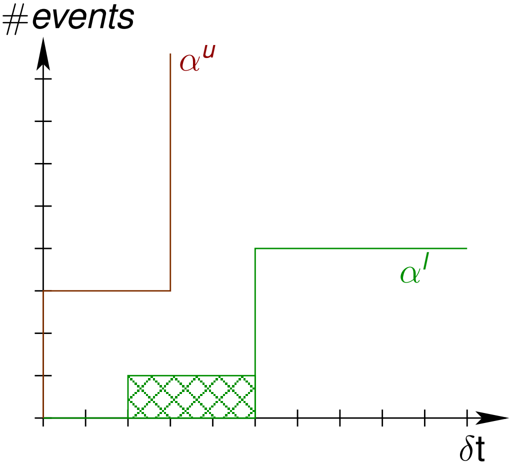

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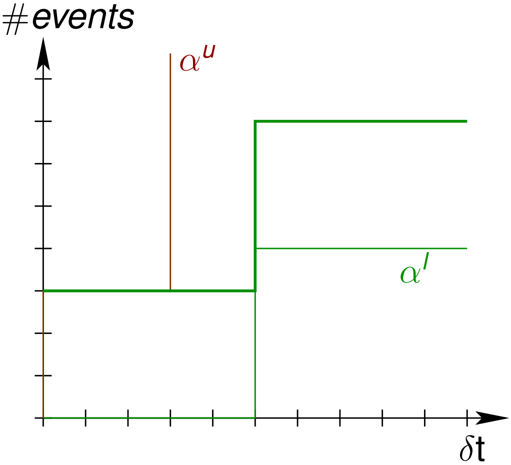

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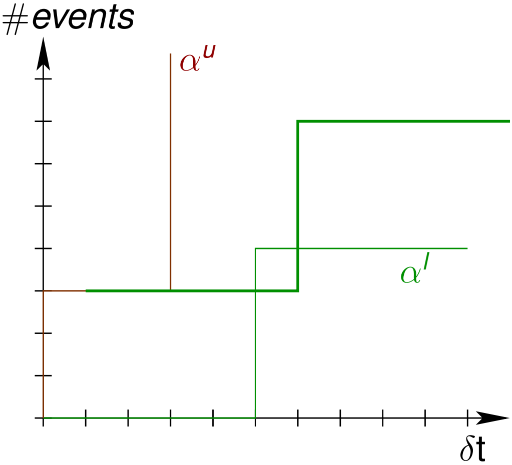

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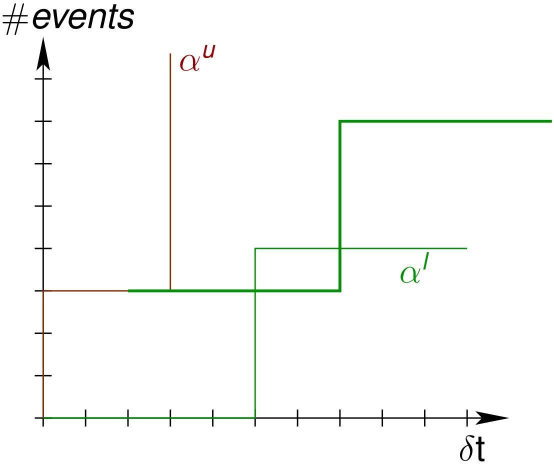

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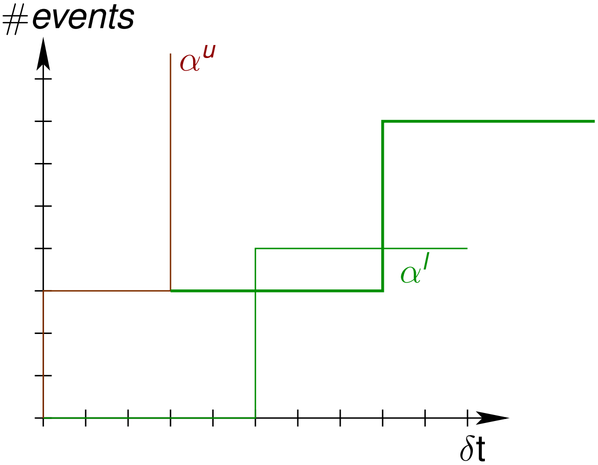

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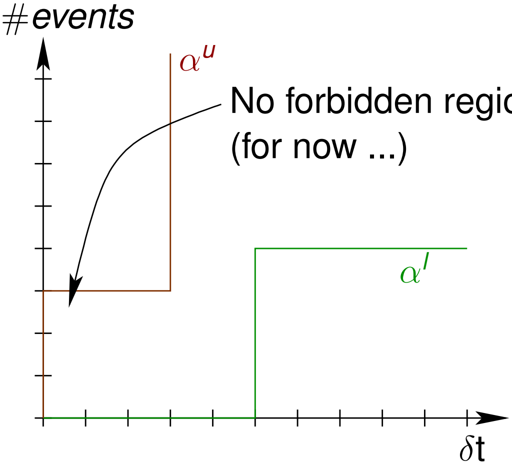

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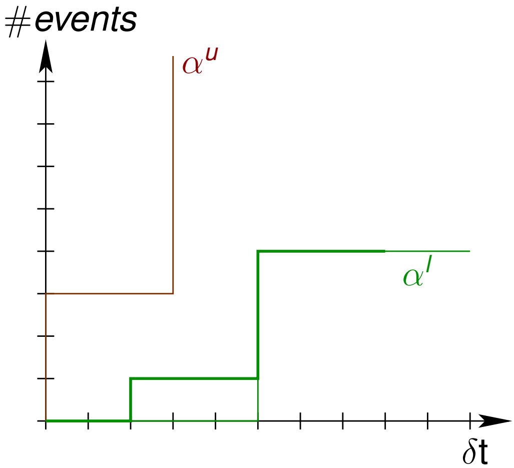

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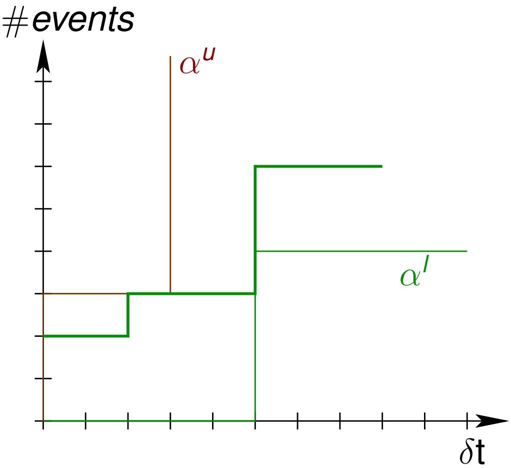

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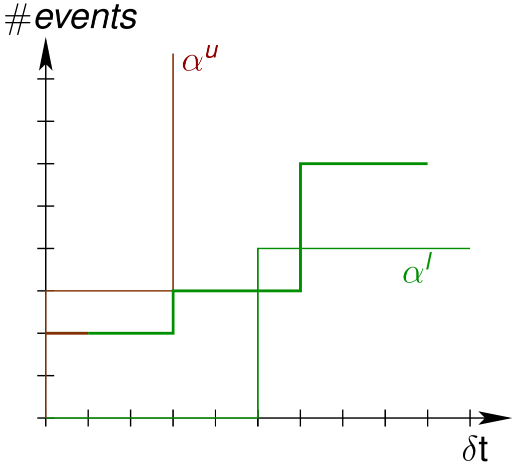

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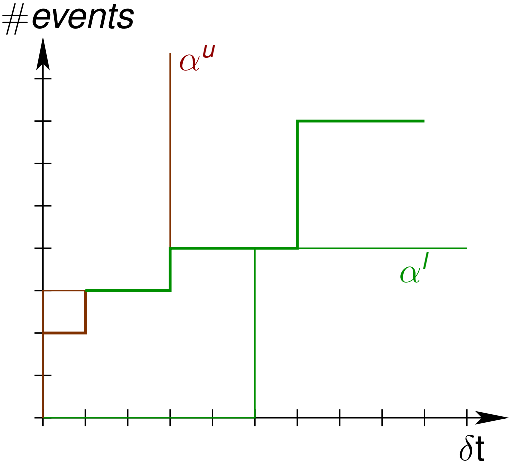

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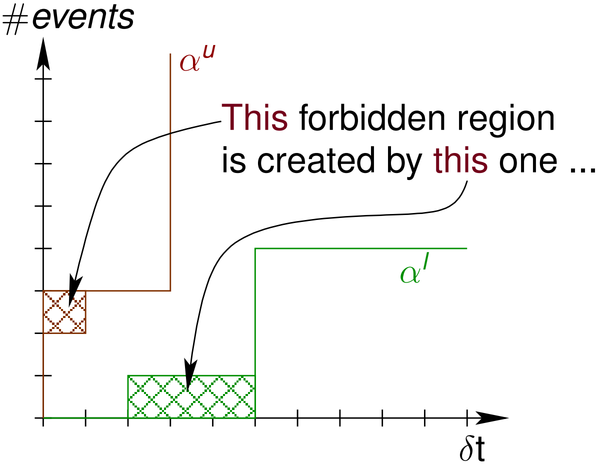

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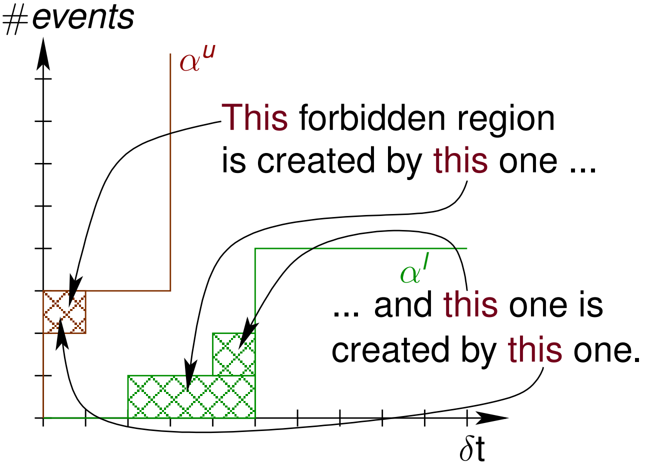

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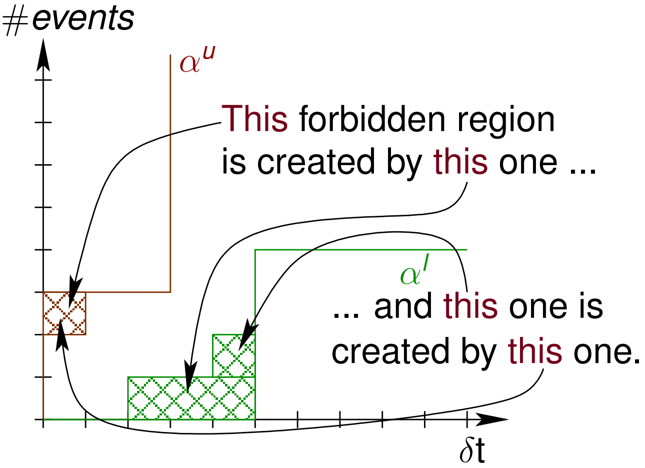

- Technically: Forbidden region removal = deconvolution = $\oslash$, $\overline{\oslash}$.

Towards an Algorithm for Causality Closure

- Second idea: Curves without forbidden regions are (?) causal, let's iterate forbidden region removal until fix-point and we're done (?)

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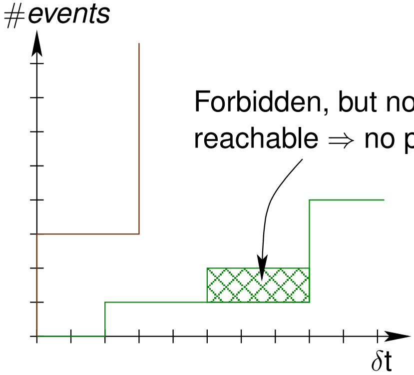
- Second idea: Curves without forbidden regions are (?) causal, let's iterate forbidden region removal until fix-point and we're done (?)
- Issue 1: Will it terminate?
- Issue 2: some causal curves do have forbidden regions!



Unreachable Regions: Another (Well Known) Issue

- $\alpha^l(\delta_1 + \delta_2)$ = minimum number of events in any time window of duration $\delta_1 + \delta_2$.
- $\alpha^l(\delta_1) + \alpha^l(\delta_2)$ is another valid bound. It may be better.
- $\Rightarrow$ If so, we say that $\alpha^l(\delta_1 + \delta_2)$ is unreachable.

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- $\alpha^l(\delta_1) + \alpha^l(\delta_2)$ is another valid bound. It may be better.
- $\Rightarrow$ If so, we say that $\alpha^l(\delta_1 + \delta_2)$ is unreachable.
- Technically, (α^u, α^l) have no unreachable regions iff
  - α^l is super-additive,
  - α^u is sub-additive.
- $\overline{\alpha^u}$ = sub-additive closure of α^u
- $\underline{\alpha^l}$ = super-additive closure of α^l

Theorem 1: Causality and Forbidden Region

For sub-additive/super-additive pair of curves,
Causality $\Leftrightarrow$ Absence of forbidden region

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$\Rightarrow$ Causal curve can be obtained by
the fix-point of forbidden region removal
for sub-additive/super-additive curves.

Theorem 2: no Need to Iterate!

- Removing forbidden regions from a sub-additive/super-additive pair of curves
  - Doesn't create new forbidden regions
  - Preserves sub-additive/super-additive property

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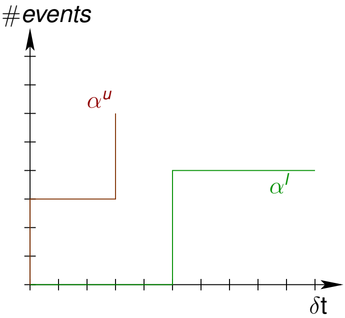
$\Rightarrow$ Applying forbidden region removal from $(\overline{\alpha^u}, \underline{\alpha^l})$ gives a causal pair of curves. This is the causality closure.

(both forbidden region removal and sub-additive/super-additive closures are cheap algorithms)

Theorem 2: no Need to Iterate!

- Removing forbidden regions from a sub-additive/super-additive pair of curves
  - Doesn't create new forbidden regions
  - Preserves sub-additive/super-additive property



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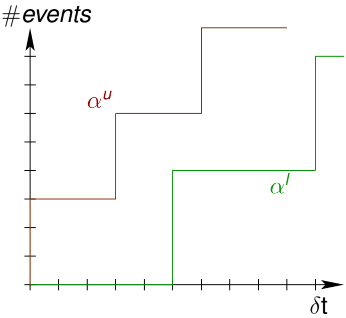

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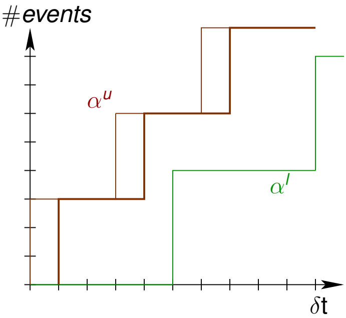

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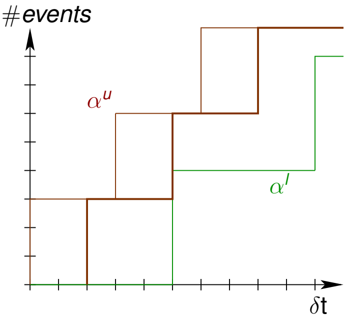

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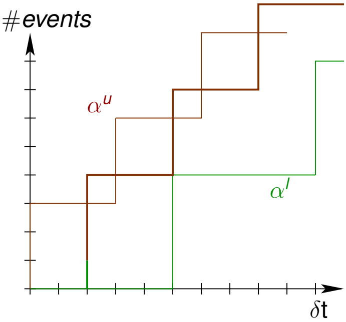

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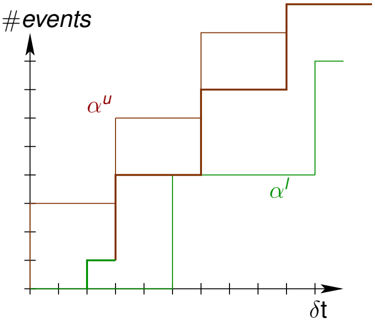

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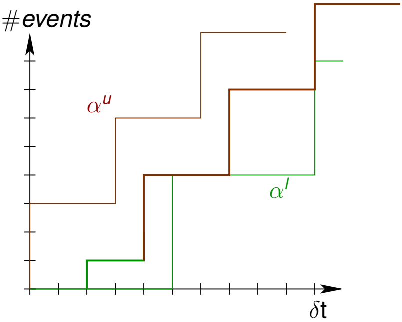

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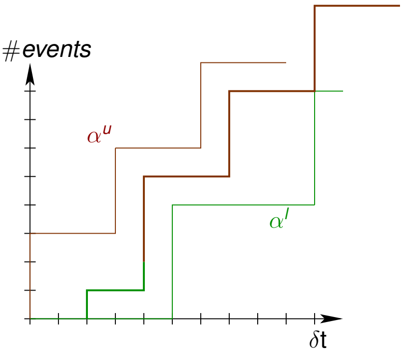

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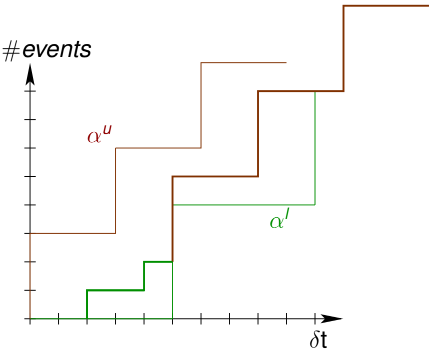

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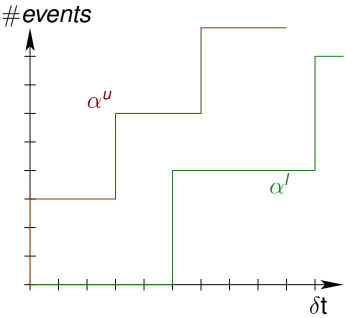

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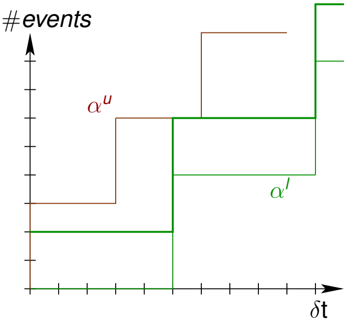

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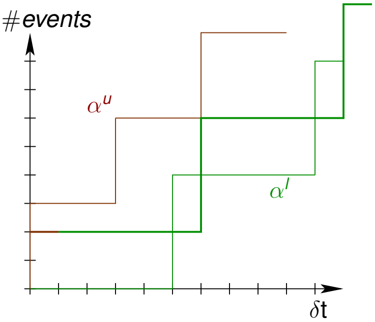

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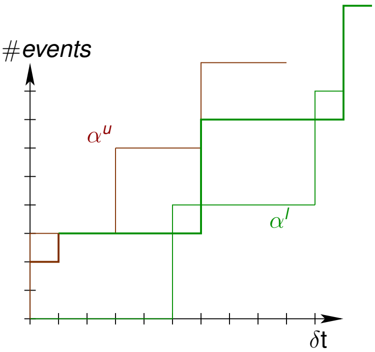

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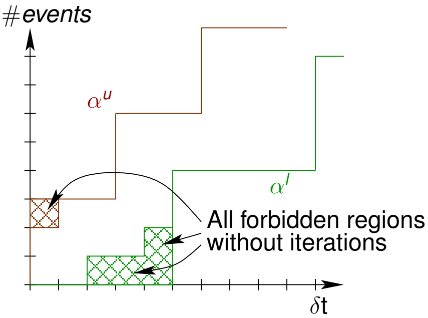

Causality Closure Algorithm

- Causality Closure Algorithm:
  1. Compute sub-additive/super-additive closure $(\overline{\alpha^u}, \underline{\alpha^l})$.
  2. Remove forbidden regions : $\overline{\alpha^l} \oslash \underline{\alpha^u}$ and $\overline{\alpha^u} \,\overline{\oslash}\, \underline{\alpha^l}$.
- Implementable in any framework implementing $\overline{\alpha}$, $\underline{\alpha}$, $\oslash$ and $\overline{\oslash}$.

Theorem 3: Optimality

Applying forbidden region removal from $(\overline{\alpha^u}, \underline{\alpha^l})$ gives the tightest pair of curves equivalent to (α^u, α^l)

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Applying forbidden region removal from $(\overline{\alpha^u}, \underline{\alpha^l})$ gives the tightest pair of curves equivalent to (α^u, α^l)

$\Rightarrow$ Give me a pair of curves,
and I'll give you a better one,
(almost) for free!

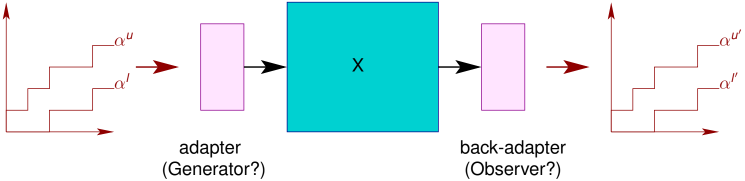
Corollary 4: Converse of Optimality

(Reminder: Theorem 3 = Causality closure is optimal)

Conversely, the tightest pair of curves is causal and sub-additive/super-additive.

$\Rightarrow$ optimal computations do not have the causality problem.
Non-causal curves are caused by over-approximations.

Connection from RTC to X, Revisited



- Compute the causality closure of (α^u, α^l) beforehand $\Rightarrow$ avoids deadlocks in the generator (and spurious counter-examples in proofs)
- Compute the causality closure of $(\alpha^{u'}, \alpha^{l'})$ afterwards $\Rightarrow$ may increase precision
- (Same applies for service curves)

Summary

Summary of Contributions

- Identification and formalization of the causality problem in RTC,
- Causality closure: make curves causal/remove deadlocks with a cheap algorithm,
- Interesting side-effect: optimality.
- Implementation for finite, discrete curves.

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Future Works

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Problem solved, move to another one ;-)

Future and Related Works

Problem solved, move to another one ;-)

- Work on connection of RTC to other formalisms :
  - Lustre
  - Timed Automata (QAPL talk on Sunday)
  - $\Rightarrow$ Causality closure is definitely useful there.
- Define causality closure for more classes of curves (mixed discrete curves + piecewise affine)

Details I've spared you ...

- Algorithm for finite, discrete curves
  (which do need iterations)
- Proofs
  (surprisingly tricky, full paper is 28 pages mostly in LaTeX math mode!)

Questions?

Backup Slide

$$
\begin{array}{ccc}
\begin{array}{c} \alpha^{l} = \alpha^{l} \oslash \alpha^{u} \\ \text{and} \\ \alpha^{u} = \alpha^{u} \mathbin{\overline{\oslash}} \alpha^{l} \end{array} & \Longrightarrow & (\alpha^{u}, \alpha^{l}) \text{ is causal} \\
\\
\Downarrow & \Nearrow & \Updownarrow \\
\\
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